FLYPING CONJECTUR

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ABSTRACT: : *In this paper we will establish the Tait's flyping conjecture "two reduced alternating knots are equivalent iff they can be converted into one another by flypes" by generalizing Reidemeister moves especially Reidemeister move of type II and therein its consequent applications.*

Keywords: Reidemeister moves, Dual graphs, ambient isotopy invariants, reduced alternating knots , equivalent knots .

Mathematics Subject Classification 2000: 57M25(15)(27)

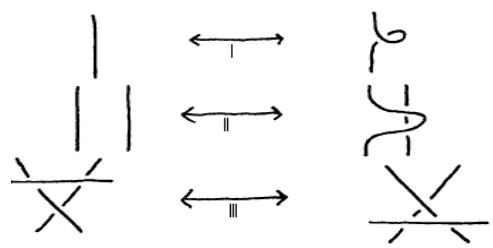
INTRODUCTION

Mathematicians were perplexed at the seemingly unending number of ways a knot could be shaped and turned. Consequently, these give rise to the central problem of knot theory i.e., whether two knots are equivalent or not. This was the motivation for much of the recent work in knot theory, which is devoted to search for invariants of knots. Reidemeister moves, tricoloring, knot polynomials (Alexander polynomial, Jones polynomial, Bracket polynomial, Homfly polynomial, Kauffman polynomial, etc.) are few examples. The study of invariants underwent in a kind of phase transition, which has linked knot theory to chemistry, molecular chemistry, mathematical physics, particles physics, polymer physics, statistical mechanics, fluid mechanics, kinematics, C^*-algebra, conformal field theory, crystallography, cryptography, graph theory, computer systems and networks, etc. In the recent past, biologists and chemists studying genetics discovered an exciting link of knot theory with DNA (genetic material of all cells, containing coded information about cellular molecules and processes) and synthetic chemistry. DNA is just one application of knot theory, which presently is an area of intense mathematical activities worldwide.

P.G. Tait studied knots in response to Kelvin's conjecture that the atoms were composed of knotted vortex tubes of ether [6]. He categorized knots with respect to crossings of their projections. He made many conjectures, among them was flyping conjecture which states that the number of crossings is the same for any reduced projection of an alternating knot or equivalently "Two reduced alternating knots are equivalent iff they can be converted into one another by flypes" The conjecture was proved by Menasco and Thistlewaite [3,4] by using Jones Polynomial. Azram [1] has proved that the number of crossings in a reduced alternation knot is topological as well as ambient isotopy invariant. In this paper we will prove the Tait's flyping conjecture by generalizing Reidemeister moves [5] especially Reidemeister move of type 2 and therein its consequent applications

MATERIAL AND METHODS

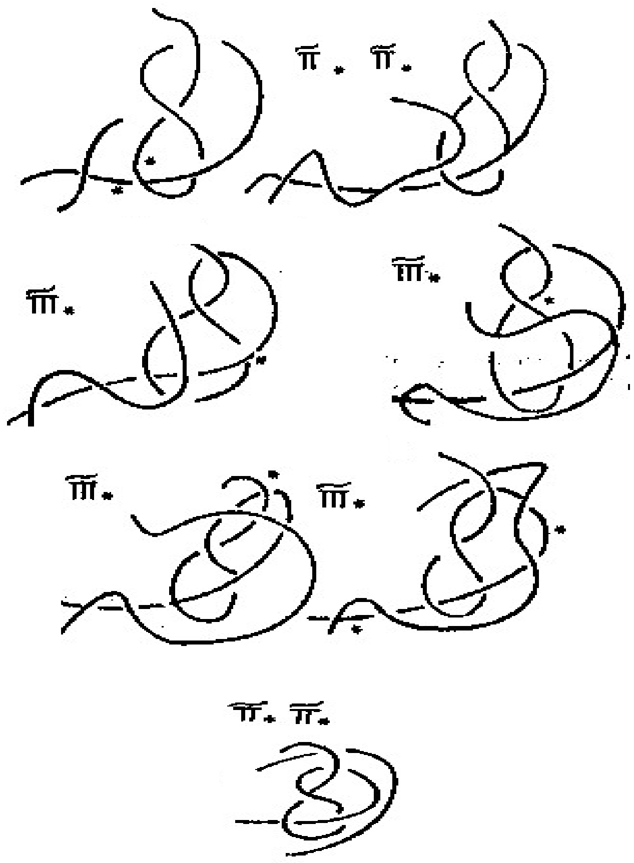
In the future, knots will be confused with their class of projections with crossings indicated unless otherwise stated. Terminology, definition and concepts for most of the material are standard. By the planar isotopy we mean the motion of the knot projection in the plane that preserves the graphical structure of the underlying universe. The pivotal moves in the theory of knots are the Reidemeister moves. We will view these moves as Reidemeister moves of type I, II, and III.



Two knots in space can be deformed into each other (ambient isotopy) if and only if their projections can be transformed into one another by planar isotopy and the three Reidemeister moves. Two knots are equivalent (via Reidemeister moves) denoted by the symbol ~, if and only if (any of) their projections differ by a finite sequence of Reidemeister moves [4]. Ambient isotopy and equivalence via Reidemeister moves is the same [5]. A crossing will be isthmus if any two of the four local regions at the crossing are the same. A knot (link) with no isthmus crossing is called reduced**.** An alternating knot is a knot where the crossing alternate under-over-under, **---**as one travels along each component of the knot, crossing at all crossings.

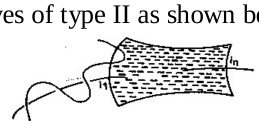
RESULTS AND DISCUSSION

The results and discussion in this article are variational, diagrammatic and illustrative, suspecting a new direction to prove Tait flyping conjecture. The R^*-move, which is a generalized form of Reidemeister move of type II, will be a pivotal move in the discussion hereafter. Before establishing R^*-move, let us consider the following example. Note that by $\underset{a*}{\sim}$ we mean that a move of type '•' is performed at the location '*'.

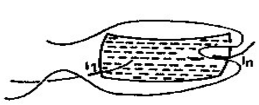


Theorem 1. R*-move is well defined via Reidemeister moves, that is,

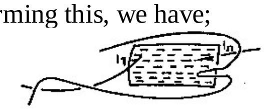
Proof. Without loss of generality, assume the part of the knot (link) has n number of crossings. Let i_1 be the very first crossing encountered while going from left to right and i_n be the very last one. Performing a couple of Reidemeister moves of type II as shown below, we have;



Now, the crossing i_1 is a candidate for a Reidemeister move of type III. We perform it and continue performing a finite sequence of suitable Reidemeister moves over all the crossings falling between the crossing i_1 and i_n, resulting as;

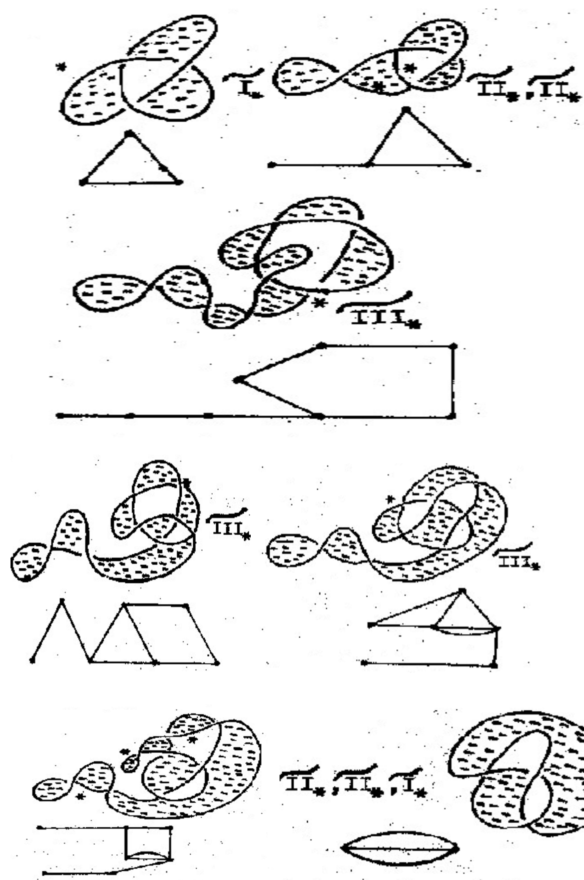


Now, the crossing i_n is a candidate for Reidemeister move of type III; performing this, we have;



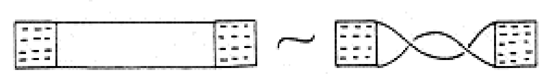
Now, perform a couple of Reidemeister moves of type II; one can easily achieve as required.■

Now, let us observe the effects of R*-move on the corresponding graph of the knot.

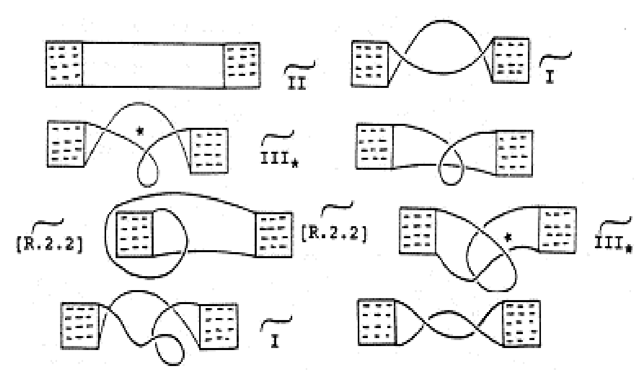


Observe that R*-move changes the black regions into white regions and vice versa. The graph corresponding to black regions systematically changed to the graph that corresponds to the white regions of the same knot producing the equivalent dual graphs. If we consider the labeled graph then the labeling (crossings) also changes systematically and accordingly.

Theorem 2. A 2Π-Twist can be established via Reidemeister moves. i.e.

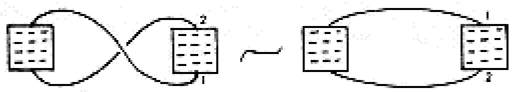


Proof.

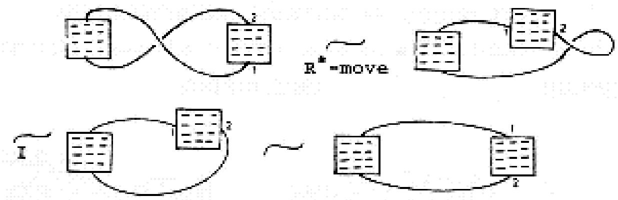


As required. This completes the proof■

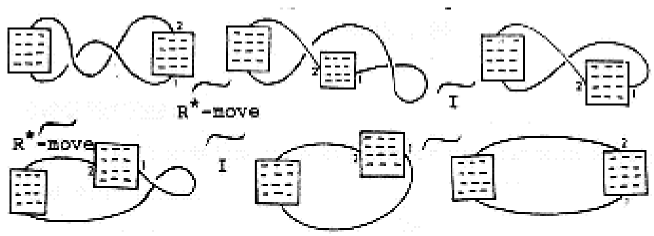
Theorem 3. A Π-Twist can be established via via Reidemeister moves. i.e.



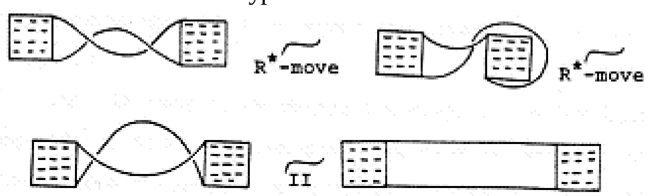
Proof. It is just the consequence of R*-move and a Reidemeister move of type I. For details, see the following;



Note. A 2Π-Twist can also be established via R*-move and a Reidemeister move of type I i.e.



Note. A 2Π-Twist can also be established via R*-move and a Reidemeister move of type II i.e.



Note. A Π-twist move eliminates the asthmus crossings and vice versa.

A crossing will be isthmus if any two of the four local regions at the crossing are the same or equivalently if each crossing has four distinct regions. Since a Π-twist move eliminates the asthmus crossings and vice versa. So consequently we can say;

Lemma. Each connected knot(link) has a reduced knot(link)

Theorem 4. Tait flyping conjecture i.e. Two reduced alternating knots are equivalent iff they can be converted into one another by flypes.

Proof. Consequence of R*-move and Π-Twist move.

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